Problem Set # 1

(On Newtonian Relativity)

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> *"Mathematics reveals its secrets only to those who approach it with pure love, for its own beauty."*
>
> — Archimedes

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1. Show that momentum conservation principle in a collision of two objects is valid in all inertial frames.

   (**Hint**: Show that in both inertial frames total momentum before collision is equal to total momentum after collision under the Galilean transformation.)

2. Using Galilean transformation show that kinetic energy conservation principle in an elastic collision of two bodies is same for all inertial frames.

3. Jacqueline is riding on her horse at a velocity of 10 m/s. She twists around in her saddle and fires a bullet backward. Her gun fires bullets at a velocity of 150 m/s. Find the speed of the bullet relative to the ground.

4. Show that Newton's second law has rotational symmetry, i.e., under rotation of coordinate axis with some angle ϕ it is covariant.

   (**Hint**: Take a set of axis xyz (say S frame) rotate it with respect to z axis with an angle ϕ. Now you have a rotated frame say S' frame with a set coordinate axis $x'y'z'$. Next, obtain the transformation relation between the coordinates of S frame to S'

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frame and use it to obtain rotation matrix for velocity vector and acceleration in the rotated frame in terms of the quantities of old frame.)

5. Recall Maxwell's equations [‡] in free space

$$\nabla.\mathbf{E} = 0, \tag{1}$$

$$\nabla.\mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{4}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields respectively. The quantity $c = 1/\sqrt{\epsilon_0\mu_0}$ and ∇ is derivative operator called del operator given by

$$\nabla \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}.$$

Now taking curl on both sides of Eq. (3), we obtain

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla \times \mathbf{B}). \tag{5}$$

Next, we use Eq. (4) in above equation and obtain

$$\nabla(\nabla.\mathbf{E}) - \nabla^2\mathbf{E} = -\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}. \tag{6}$$

Using Eq. (1) for $\nabla.\mathbf{E} = 0$, we obtain

$$\nabla^2\mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}. \tag{7}$$

Similarly, for the field $\mathbf{B}$, we obtain equation in similar form

$$\nabla^2\mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}. \tag{8}$$

Now both the Eqs. (7) and (8) can be written in compact form as

$$\nabla^2\mathbf{\Phi} = \frac{1}{c^2}\frac{\partial^2 \mathbf{\Phi}}{\partial t^2}, \tag{9}$$

where $\mathbf{\Phi}$ denotes the fields either $\mathbf{E}$ or $\mathbf{B}$. In three spatial dimensions, the quantity $\mathbf{\Phi}$ has three component Φ_x, Φ_y and Φ_z. We call Eq. (9) as the electromagnetic wave equation. We can rewrite Eq. (9) as

$$\frac{\partial^2 \mathbf{\Phi}}{\partial x^2} + \frac{\partial^2 \mathbf{\Phi}}{\partial y^2} + \frac{\partial^2 \mathbf{\Phi}}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \mathbf{\Phi}}{\partial t^2}. \tag{10}$$

[‡] See any standard book on electrodynamics, e.g., *Introduction to Electrodynamics* by David J. Griffiths

Here is your task: Use Galilean transformation rule and see if Eq. (10) is invariant or not.

Hint: The Galilean transformation is given by

$$x' = x - ut,\ y' = y,\ z' = z;\ \ t' = t. \tag{11}$$

Now we write

$$\frac{\partial}{\partial x} = \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} + \frac{\partial y'}{\partial x}\frac{\partial}{\partial y'} + \frac{\partial z'}{\partial x}\frac{\partial}{\partial z'}. \tag{12}$$

From Eq. (11), we see that

$$\frac{\partial x'}{\partial x} = 1,\ \frac{\partial y'}{\partial y} = 1,\ \frac{\partial z'}{\partial z} = 1,\ \frac{\partial t'}{\partial t} = 1,\ \frac{\partial x'}{\partial t} = -u. \tag{13}$$

Now using Eq. (13) in (12), we obtain

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}. \tag{14}$$

Similarly,

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y'},\ \frac{\partial}{\partial z} = \frac{\partial}{\partial z'}. \tag{15}$$

Again

$$\frac{\partial}{\partial t} = \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} + \frac{\partial y'}{\partial t}\frac{\partial}{\partial y'} + \frac{\partial z'}{\partial t}\frac{\partial}{\partial z'} + \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} \tag{16}$$

$$= -u\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}. \tag{17}$$

Similarly, obtain transformations in second derivatives and see if the form of Eq. (10) remains same or changes under the Galilean transformation and hence write your comment on the final result.